Published by Institute of Physics Publishing for SISSA/ISAS
Received: *May 5, 2004*
Revised: *May 26, 2004*
Accepted: *June 10, 2004*

Witten-Veneziano from Green-Schwarz

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Abstract: We consider the U(1) problem within the AdS/CFT framework. We explain how the Witten-Veneziano formula for the η' mass is related to a generalized Green-Schwarz mechanism. The closed string mode, that cancels the anomaly of the gauged U(1) axial symmetry, is identified with the η' meson. In a particular set-up of D3-branes on a $\mathbb{C}^3/(\mathbb{Z}_3 \otimes \mathbb{Z}_3)$ orbifold singularity, the η' meson is a twisted-sector R-R field.

Keywords: 1/N Expansion, AdS-CFT and dS-CFT Correspondence, Anomalies in Field and String Theories.

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 http://jhep.sissa.it/archive/papers/jhep062004019/jhep062004019.pdf

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1. Introduction

The U(1) problem was considered as one of the major issues in particle physics in the 70's. The dynamical breaking of the $\mathrm{U_L}(3) \times \mathrm{U_R}(3)$ chiral symmetry to the diagonal $\mathrm{U_V}(3)$ should result in nine Goldstone bosons, whereas only a light octet is observed in nature. The η' meson that correspond to the breaking of the $\mathrm{U_A}(1)$ symmetry is too heavy to be a Goldstone.

It was understood that, at the qualitative level, the resolution of the puzzle should involve the ABJ anomaly. It was later suggested by 't Hooft that the η' meson becomes massive because of instantons [1]. A different solution, which will be reviewed here, was suggested by Witten [2] and by Veneziano [3]. They showed that the η' should be massless in the 't Hooft large-N limit (the planar theory), since the anomaly is a $1/N$ effect, and that a $1/N$ mass for the η' is generated when the anomaly is taken into account. Their analysis resulted in the celebrated "Witten-Veneziano formula" (WV) [2, 3]:

$$M^2_{\eta'} = \frac{4N_f}{f_\pi^2}\frac{1}{(16\pi^2)^2}\int d^4x\, \langle \mathrm{tr}\, F\tilde{F}(x)\, \mathrm{tr}\, F\tilde{F}(0)\rangle|_{N_f=0}\,. \tag{1.1}$$

In this note we would like to derive the WV formula (1.1) from the AdS/CFT correspondence [4]. Aspects of chiral symmetry breaking within the "AdS/CFT with flavor" approach were discussed recently in [5].[1] As we shall see the derivation links the WV formula to the Green-Schwarz (GS) mechanism [6]: the closed string that is needed to cancel the $\mathrm{U}_A(1)$ anomaly will be identified with the η' meson, see figure 1.

The set-up that we will use is D3-branes on orbifold singularities. Although we will use a specific orbifold, we suggest that this phenomenon is common to a generic set-up of D-branes on orbifold singularities. In particular it does not require supersymmetry. In fact, it should be even more general: in string theory there are no global symmetries. If the set-up involves the field-theory U(1) axial symmetry as a symmetry of string theory,[2] it must be local. Local symmetries cannot be anomalous and hence the anomaly must be

[1] Note that in those models the η' is massless, since the limit $N_f/N_c \to 0$ is assumed.

[2] The U(1) axial symmetry might be an "accidental symmetry" as well.

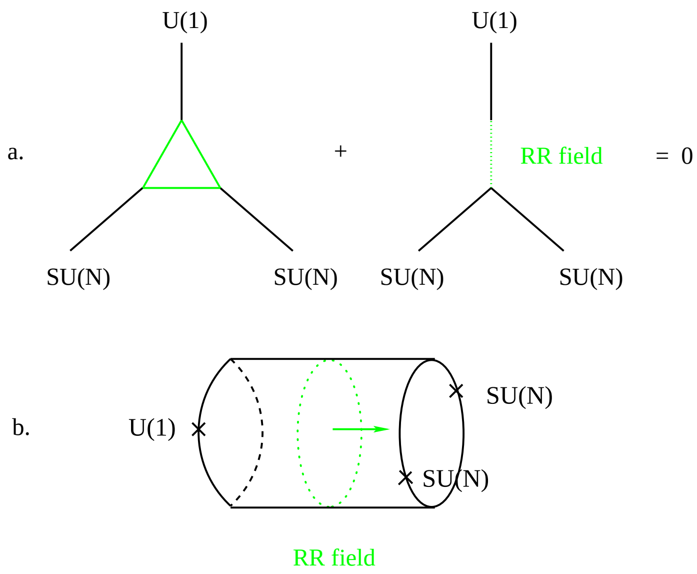

Figure 1: (a) The Green-Schwarz mechanism. (b) The string theory diagram: the R-R closed string mode is identified with the η' meson.

canceled via a generalized GS mechanism. The intermediate (odd parity R-R) closed string mode, which is involved in the anomaly cancellation, is the η' meson. Another remark is that we are taking the standard AdS/CFT decoupling limit: $\alpha' \to 0$, while keeping R^2/α' fixed. The proposed GS mechanism "survives" this limit, as the anomaly is a field theory effect.

While our analysis is general, it would be interesting to make an explicit calculation of the η' mass in the resolved $\mathbb{C}^3/(\mathbb{Z}_3 \otimes \mathbb{Z}_3)$ orbifold model (in units of the string tension).

The rest of this note is organized as follows: in section 2 we describe our model and in section 3 we show how, within our specific set-up, we can derive the WV formula.

2. D3 branes on $\mathbb{C}^3/(\mathbb{Z}_3 \otimes \mathbb{Z}_3)$ orbifold singularity

In this section we set the general framework. Consider, as in [7], the type-IIB string with a stack of N D3-branes placed on a $\mathbb{C}^3/\Gamma$ orbifold singularity. In particular we will be interested in the $\mathbb{C}^3/(\mathbb{Z}_3 \otimes \mathbb{Z}_3)$ orbifold. This model, as well as the general case of D3-branes on $\mathbb{C}^3/(\mathbb{Z}_k \otimes \mathbb{Z}_{k'})$ orbifold singularities, were analyzed in [8].

The conformal field theory on the D3-branes is a supersymmetric chiral $\mathrm{U}^9(N)$ gauge theory with bi-fundamental matter. Therefore, there are mixed anomalies of the form $\mathrm{U}_i(1)$ $\mathrm{SU}_j^2(N)$ (in fact, some combinations of U(1)'s are anomaly-free).

The resolution of this local anomaly problem in the $\mathbb{C}^3/(\mathbb{Z}_3 \otimes \mathbb{Z}_3)$ model is via a generalized GS mechanism [9]. The anomalous U(1)'s receive a mass of the string scale and therefore they are infinitely massive from the field theory point of view. The low-energy gauge group is thus $\mathrm{SU}^9(N)$ (plus additional anomaly-free U(1)'s). In addition the local anomalies are canceled via an exchange of massless closed strings, see figure 1. In the above model the closed strings modes are twisted-sector R-R fields. The terms that give mass to the U(1)'s and cancel the anomaly are WZ-terms, which are localized on the world-volume of the D3-branes:

$$\int d^4x \; C \wedge \exp F \,. \tag{2.1}$$

Indeed, the result of the detailed analysis of ref. [8] is that there are six R-R (and NS-NS) twisted-sector moduli. They live on $\mathbb{C} \in \mathbb{C}^3$, hence they are six-dimensional.

In the AdS/CFT framework (the near-horizon limit of the configuration), we consider type-IIB string theory on $AdS_5 \times S^5/(\mathbb{Z}_3 \times \mathbb{Z}_3)$. The R-R twisted sector moduli that cancel that U(1) anomalies live on $AdS_5 \times S^1$.

The above model is satisfactory for most of our purposes; however, in order to have a clearer picture, we would still like to make two simplifications. We wish to turn off the coupling of all but one local $\mathrm{U}(N_c)$ group. In the string theory set-up, combinations of gauge couplings are controlled by NS-NS moduli. By varying the six-dimensional twisted NS-NS moduli, we can achieve the limit where the coupling of eight of the nine gauge factors is much smaller than the coupling of the ninth. This is clearly seen in the T-dual picture [8].

The second simplification that we wish to make is to change the rank of the symmetry group so that we will have a $\mathrm{U_L}(n_f) \times \mathrm{U}(N_c) \times \mathrm{U_R}(n_f)$ group. String theory allows us to do so by adding fractional branes at the orbifold singularity. A different way of achieving an arbitrary number of flavors, including the situation with no flavors at all, is to start with the T-dual picture of D5-branes and NS5-branes and to vary the number of D5-branes [8]. Thus we will consider the model in table 1.

	$\mathrm{U_L}(n_f)\times$	$\mathrm{U}(N_c)\times$	$\mathrm{U_R}(n_f)$
3 chirals	$\square$	$\bar{\square}$	1
3 chirals	1	$\square$	$\bar{\square}$

Table 1: The simplified model: a $\mathrm{U}(N_c)$ gauge theory with fundamental and anti-fundamental matter.

Since the two groups $\mathrm{U_L}(1)$ and $\mathrm{U_R}(1)$ are global (or very weakly coupled), the twisted-sector R-R fields will not lift their masses. Only the mass of the center of the $\mathrm{U}(N_c)$, which can be identified with the $\mathrm{U_B}(1)$ baryon number, is lifted by the mixed anomalies $\mathrm{SU}^2_{\mathrm{L}}(n_f)\,\mathrm{U_B}(1)$ and $\mathrm{SU}^2_{\mathrm{R}}(n_f)\,\mathrm{U_B}(1)$. Thus the model we consider here is a $\mathrm{SU}(N_c)$ gauge theory with $N_f \equiv 3n_f$ fundamental fermions (and scalars) and N_f anti-fundamentals fields. Actually, when all (but the gauge) interactions are turned off, the global symmetry is enhanced to $\mathrm{SU}(N_f)$. The GS action, in terms of two R-R fields, denoted by C_{L} and C_{R}, takes the following form (see ref. [10] for an explicit derivation)

$$S = \int d^4x \frac{1}{\alpha'}\left(\frac{1}{2}(\partial C_{\mathrm{L}} - \partial C_{\mathrm{R}})^2 + \frac{1}{2}(\partial C_{\mathrm{L}} + \partial C_{\mathrm{R}} - \mathrm{tr}\, A)^2\right) + \int d^4x\, \frac{N_f}{8\pi^2}(C_{\mathrm{L}} - C_{\mathrm{R}})\,\mathrm{tr}\, F\tilde{F}\,. \tag{2.2}$$

Note that the anomaly is proportional to N_f. The action $C_{\mathrm{L}} \to C_{\mathrm{L}} + \alpha_{\mathrm{L}}$ ($C_{\mathrm{R}} \to C_{\mathrm{R}} + \alpha_{\mathrm{R}}$) corresponds to a chiral rotation of the fundamental (anti-fundamental) fermions. A somewhat simpler way of writing (2.2) is in terms of the combinations $C_{\mathrm{A}} \equiv C_{\mathrm{L}} - C_{\mathrm{R}}$ and $C_{\mathrm{V}} \equiv C_{\mathrm{L}} + C_{\mathrm{R}}$

$$S = \int d^4x\, \frac{1}{\alpha'}\left(\frac{1}{2}(\partial C_{\mathrm{A}})^2 + \frac{1}{2}(\partial C_{\mathrm{V}} - \mathrm{tr}\, A)^2\right) + \frac{N_f}{8\pi^2} C_{\mathrm{A}}\,\mathrm{tr}\, F\tilde{F}\,. \tag{2.3}$$

The symmetry $C_{\rm A} \to C_{\rm A} + \alpha$, which became global once the gauge couplings of the $\mathrm{U_L}(n_f)$, $\mathrm{U_R}(n_f)$ gauge groups were turned off, is the axial symmetry. Note the similarity of (2.3) to the QCD effective action for the η' [11] (see also [12]), upon the identification $C_{\rm A} = -\eta'$.

3. A derivation of the Witten-Veneziano formula

We now wish to derive the WV formula (1.1) from the AdS/CFT correspondence. In the AdS/CFT framework, color singlets are described by closed strings [13]. We suggest that the η' meson of the $\mathbb{C}^3/(\mathbb{Z}_3 \otimes \mathbb{Z}_3)$ model, namely the lightest pseudo-scalar meson, is the twisted R-R field $C_{\rm A}$. The arguments in favor of this suggestion are clear: the action $C_{\rm A} \to C_{\rm A} + \alpha$ is the generator of the axial symmetry. Moreover, the field $C_{\rm A}$ is a pseudo-scalar and it couples to the anomaly with a strength N_f/N_c, as we expect from the η'.[3] Finally, we will show below that the mass of the $C_{\rm A}$ field admits the WV formula (1.1).

We start by an evaluation of the two-point function $\langle \mathrm{tr}\, F\tilde{F}(x)\, \mathrm{tr}\, F\tilde{F}(0)\rangle$. The AdS/CFT prescription for calculating two-point functions $\langle O(x)\, O(0)\rangle$ in the boundary field theory is to evaluate the propagators of the bulk closed strings that couple to the boundary operator $O(x)$ [4]. There are two bulk fields that couple to $\mathrm{tr}\, F\tilde{F}$: the twisted sector R-R field $C_{\rm A}$ (2.2) and the ten dimensional R-R 0-form C (the "axion"). In fact, it is well known that the axion describes pseudo-scalar glueballs [14]. Thus, according to the AdS/CFT correspondence

$$\frac{1}{(16\pi^2)^2}\langle \mathrm{tr}\, F\tilde{F}(x)\, \mathrm{tr}\, F\tilde{F}(0)\rangle = \xi^2 \sum_{\mathrm{C\,modes}} K_C(x,0) + \lambda^2 \sum_{\mathrm{C_A\,modes}} K_{C_{\rm A}}(x,0)\,, \tag{3.1}$$

where ξ, λ are the couplings of the ten-dimensional R-R 0-form and the twisted sector R-R 0-form to $O = \mathrm{tr}\, F\tilde{F}$, respectively. $K(x,0)$ is a boundary-to-boundary propagator. In the following we will identify the C-modes and the $C_{\rm A}$-modes with glueballs and flavor singlet mesons respectively.

The propagator of a massless bulk field in a *confining* supergravity background is the same as that of a free massive field in a flat space [13]. It is seen by taking the ansatz $K(x;r) = e^{iqx}F(r)$ and by solving the 5-d bulk wave equation. Indeed, for a background of the form

$$ds^2 = dr^2 + f(r)dx_\mu^2\,, \tag{3.2}$$

The bulk wave-equation takes the form

$$f^{-1}(r)\partial_r\left(f^2(r)\partial_r F(r)\right) - q^2F(r) = 0\,. \tag{3.3}$$

The eigenvalues q^2 are interpreted as the 4-d mass of the color-singlet hadron and thus the boundary-to-boundary propagators, in momentum space, take the form

$$K_C = \frac{1}{q^2 - M_C^2}\,; \qquad K_{C_{\rm A}} = \frac{1}{q^2 - M_{C_{\rm A}}^2}\,. \tag{3.4}$$

[3] In our set-up, the ratio N_f/N_c is kept fixed, while $N_c \to \infty$; the η' is therefore expected to have a non-vanishing mass (in contrast to the set-ups in refs. [5]).

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Since the theory contains massless quarks[4]

$$\int d^4x \, \langle \mathrm{tr}\, F\tilde{F}(x) \, \mathrm{tr}\, F\tilde{F}(0) \rangle = 0 \, . \tag{3.5}$$

(There is no meaning to a theta vacuum, since we can use a chiral rotation to set the value of theta to an arbitrary value). Let us proceed as in [2]. In the version of our theory without matter, namely when $N_f/N_c = 0$, the quantity $\int d^4x \, \langle \mathrm{tr}\, F\tilde{F}(x) \, \mathrm{tr}\, F\tilde{F}(0) \rangle$ is non-vanishing. We therefore must conclude that in eq.(3.1) the contribution from the mesons cancels that from the glueballs:

$$\frac{1}{(16\pi^2)^2} \int d^4x \, e^{iqx} \, \langle \mathrm{tr}\, F\tilde{F}(x) \, \mathrm{tr}\, F\tilde{F}(0) \rangle |_{\frac{N_f}{N_c}=0, q^2=0} = \frac{\lambda^2}{M^2_{C_A}} \, . \tag{3.6}$$

In fact, when $N_f/N_c \to 0$ the sum over mesons (C_{A}-modes) is dominated by the lightest mode whose mass tends to zero since in this limit the anomaly vanishes and C_{A} becomes a true Goldstone. In this limit C_{A} does not couple to the boundary field theory and (3.3) is solved by $F(r) = \mathrm{const}$.

The second ingredient that is needed to complete our derivation is the anomaly equation. From (2.3) we can read the equation of motion for C_{A}

$$\frac{1}{\alpha'} \partial_\mu \partial^\mu C_{\mathrm{A}} = \frac{N_f}{8\pi^2} \, \mathrm{tr}\, F\tilde{F} \, . \tag{3.7}$$

We therefore identify $\frac{1}{\alpha'}\partial^\mu C_{\mathrm{A}}$ with the axial current J^μ. The coupling of the r.h.s. of (3.7) to C_{A} is by definition $2N_f\lambda$. Following ref. [2] we argue that the coupling of $\partial_\mu J^\mu$ to C_{A} is proportional to $M^2_{C_{\mathrm{A}}}$, with a proportionality parameter f_{A}. The reason is that, in momentum space, the coupling of J^μ should be $\sim q^\mu$ (the momentum is the only vector in the problem) and therefore the coupling of $q_\mu J^\mu$ is $\sim q^2 = M^2_{C_{\mathrm{A}}}$. Thus

$$2N_f\lambda = f_{\mathrm{A}} M^2_{C_{\mathrm{A}}} \, . \tag{3.8}$$

Inserting (3.8) in (3.6)

$$M^2_{C_{\mathrm{A}}} = \frac{4N_f^2}{f_{\mathrm{A}}^2} \frac{1}{(16\pi^2)^2} \int d^4x \, e^{iqx} \, \langle \mathrm{tr}\, F\tilde{F}(x) \, \mathrm{tr}\, F\tilde{F}(0) \rangle |_{\frac{N_f}{N_c}=0, q^2=0} \, , \tag{3.9}$$

we arrive at the WV formula (1.1), provided that C_{A} is identified with η' and its coupling square f_{A}^2 with $N_f f_\pi^2$.

Note added: I have been informed by J. Barbon of a related work [15], where the WV formula is discussed within the framework of “AdS/CFT with flavor”.

Acknowledgments

I would like to thank J. Barbon, C. Hoyos, E. Lopez, A. Ritz and G. Veneziano for discussions and for comments on the manuscript.

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